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On the Variation of Personal Equation with Stellar Magnitude in Observations made at Cambridge, Berlin, and Greenwich; as deduced from Measures of Photographic Plates taken at Oxford.

By H. H. Turner, M.A., F.R.S., Savilian Professor.

The belt of the heavens now being photographed at the University Observatory, Oxford (zones +25° to +31°) as part of the work of the Astrographic Chart, includes the belt observed on the meridian at the Cambridge Observatory (+25° to +30°) during the years 1872 to 1896, as part of the revision of the D.M. according to the scheme of the *Astronomische Gesellschaft.* The places of the Cambridge Catalogue (epoch 1875·0) brought up to 1900·0 are compared with the Oxford measures to get the plate-constants; and when these have been deduced residuals are formed showing the corrections to the Cambridge places given by the Oxford measures, and are entered in ledgers.

2. There is no 'vicious circle' in first using the Cambridge places to find the Oxford plate-constants, and then using the Oxford measures corrected by these plate-constants to find the errors of individual stars in the Cambridge Catalogue; for there are many more Cambridge stars on the plate than are necessary to determine the plate-constants, for which 3 stars would suffice. There are nearly always 20 Cambridge stars on the plate, and sometimes as many as 100; and we are really finding the individual corrections for each star referred to the mean of the 20 or of the 100; just as if we observe 20 clock-stars we may determine the clock-error from the mean of them, and, using this, find the corrections to the individual assumed places.

3. The Ledgers of Corrections to Cambridge places then give us quantities made up of

(A) Proper motions between the epoch of observation at Cambridge, and the date of the plate taken at Oxford (about 1895 say).

(B) Systematic errors of the Oxford measures.

(C) Systematic Errors of the Cambridge Catalogue.

4. The present paper deals with an error which falls under head (C), viz., the variation in personal equation at Cambridge with stellar magnitude. The Oxford measures are freed from this error; for, after measuring each plate, it is rotated 180° in its own plane and re-measured. The mean of the two measures is freed from any personality or error of bisection.

[Moreover, the differences of the two measures indicates the amount of this personality, and it is not found to vary sensibly with stellar magnitude. But this is immaterial for our present purpose.]

5. Though the Ledger residuals are affected by other quantities under head (A) or (B), these may be regarded as accidental errors in the present discussion, which will disappear in the

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mean of a number of stars; and to determine the variation of personal equation with stellar magnitude at Cambridge we have little more to do than to group the residuals according to stellar magnitude and take the means of groups. A preliminary discussion was given by Mr. A. R. Hinks in 1897 (*Monthly Notices*, lvii. p. 473), but when only 7 Oxford plates had been measured. Now more than 600 plates have been measured and reduced, and it is possible to give something like a definitive character to the investigation. The matter is so important that it will no doubt be interesting to give the final results deduced from the whole 1180 plates forming the Oxford share of the work; but at the same time the quantities found are so large, and are so clearly indicated by the material already available, that it seems advisable to publish the following results at once:—

Comparison between Oxford Photographic Measures and Cambridge Transit-Circle Observations.

The sign + indicates that the Cambridge R.A. or Decl. is too small.

Mag. (Cambr.)	No. of Stars.	Corrections to Cambridge R.A.			Decl.
		O	C	O−C.	
		s	s	s	″
1·0 to 2·9	4	+·118	(+·204)	(−·086)	−0·12
3·0 to 3·9	7	+·113	(+·170)	−·057	+0·18
4·0 to 4·9	9	+·180	(+·147)	+·033	−0·39
5·0 to 5·4	38	+·131	+·129	+·002	−0·21
5·5 to 5·9	30	+·114	+·118	−·004	−0·30
6·0 to 6·4	88	+·104	+·106	−·002	−0·30
6·5 to 6·9	147	+·093	+·095	−·002	−0·27
7·0 to 7·4	320	+·084	+·083	+·001	−0·33
7·5 to 7·9	504	+·071	+·072	−·001	−0·12
8·0 to 8·4	572	+·050	+·060	−·010	−0·15
8·5 to 8·9	1226	+·023	+·022	+·001	−0·06
9·0 to 9·4	2001	−·037	−·030	−·007	+0·13
9·5 and over	438	−·066	−·070	+·004	+0·26

Column I. gives the magnitudes from the Cambridge Catalogue, Column II. the number of stars. To mag. 7·9 this is practically the total number as yet photographically observed at Oxford in all R.A.'s, but for magnitudes 8·0 to 9·5 and over only the hours 0^h—12^h R.A. have as yet been discussed. For reasons given later it is believed that the general mean would not be altered by including the whole of the faint stars, and as the present discussion is not final the additional labour has not been undertaken.

In Column III. is given the mean photographic correction to the Cambridge R.A. For the first three groups the number of

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stars is so small that the results must not be treated too seriously. Proper motions have been applied (taken from the Greenwich Catalogue 1880·0) for the interval between the Cambridge and Oxford observations, though in the other groups P.M. has been treated as accidental error.

Moreover, most of the Cambridge places for these stars do not represent actual observations, but are simply copied from Auwers' *Fundamental Catalog.* They are only Cambridge observations in this sense, that the clock-errors were derived by comparing Cambridge observations of such bright stars with the places of the *Fundamental Catalog*, so that the mean bright star of the *Fundamental Catalog* must agree with its Cambridge R.A. Of course the individual stars may show discordances, but from the comparison with Greenwich given below it seems possible that these places may be regarded sensibly as Cambridge places even for individual stars.

In Column IV. is given a theoretical correction representing the observed residuals, viz. :—

For the brighter stars to magnitude 8·0

$$\text{Correction} = +0^{s}{\cdot}25 - 0^{s}{\cdot}023 \times \text{magnitude.}$$

About magnitude 8·0 the gradient changes as indicated in the diagram, and for the fainter stars we may write

$$\text{Correction} = +0^{s}{\cdot}89 - 0^{s}{\cdot}100 \times \text{magnitude.}$$

A hyperbola would thus well represent the observations, but it seems needless to derive a more exact empirical formula.

In Column V. are shown the differences between the curve of the diagram and the observed quantities.

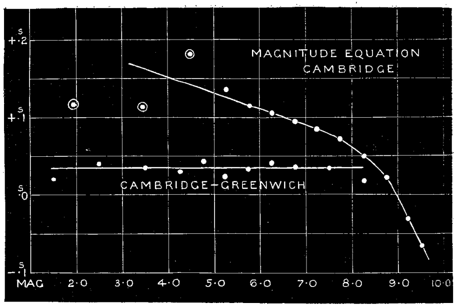

Diagram representing Variation of Personal Equation with Stellar Magnitude in Cambridge Meridian Observations.

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In Column VI. are given similar results for declination, showing that in declination there is no serious personality depending on magnitude until we reach faint stars.

Up to magnitude 8·0 the declinations are consistent with the photographic measures, neglecting the first three groups, which are not really Cambridge observations. But about magnitude 8·0 a change begins, which becomes emphatic for stars of magnitude 9·5 and over.

6. It is clear that if we would use the Cambridge meridian places to the best advantage in finding the "constants of our plates" we must apply to them :—

(*a*) Differential corrections depending on magnitude; for there is a systematic error ranging over about $0^{s}\cdot15$, even between magnitudes 8·0 and the faintest stars. On any single plate there are not likely to be more than one or two stars brighter than 8·0, but the differential correction among the more numerous faint stars is still serious.

(*b*) We must apply a considerable index error, or constant correction, to all the R.A.'s and declinations, if we wish them to be comparable with meridian observations of bright stars—say stars of the sixth magnitude. And here we are, of course, met by the difficulty of adopting a fixed standard. What observations shall we take as our starting-point?

7. A necessary preliminary to answering this question is to determine how the Cambridge observations accord with other catalogues. Mr. Hinks has given, in the paper above referred to, a comparison of Cambridge and Berlin R.A.'s, based on the common zone +24° 50′ to +25° 10′, which is practically as follows :—

Mean Mag.	Camb. R.A. —Berlin R.A.	Oxford (phot.) —Camb.	Oxford (phot.) —Berlin.
	s	s	s
6·5 ?	+ ·003	+ ·100	+ ·103
7·5	+ ·018	+ ·078	+ ·096
8·5	+ ·056	+ ·036	+ ·092
9·5 ?	+ ·093	− ·060	+ ·033

Dr. Becker's result (obtained from observations with screens which diminished the light of the star by a known amount) that his R.A.'s required correction by $-0^{s}\cdot007$ per magnitude is thus confirmed fairly well by the comparison of the first three groups. If the Berlin R.A.'s of 7·5 magnitude stars were diminished by $0^{s}\cdot007$, of 8·5 magnitude by $0^{s}\cdot014$, and of 9·5 magnitude by $0^{s}\cdot021$, the differences in the fourth column above become

Mean Mag.	Oxford photog. —Berlin corrected.
	s
6·5 ?	+ ·103
7·5	+ ·103
8·5	+ ·106
9·5 ?	+ ·054

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But there is a drop for the faint stars. Probably Dr. Becker did not push his observations far enough as regards the fainter stars. There is nothing to negative this conclusion in the summary of his observations given in the introduction to the Berlin Catalogue.

8. It thus appears that as regards the fainter stars the variation of personal equation is different for different observers. A more hopeful result, however, was obtained for the brighter stars on comparing Cambridge and Greenwich. The Greenwich Ten-Year Catalogue (1880) was compared with the Cambridge Catalogue (1875), with the following result :—

	R.A. (Cambridge)	— R.A. (Greenwich)	
Mag.	No. of Stars.	Mean Difference.	Excess over $+0^{s}{\cdot}035$.
		s	s
1·0 to 1·9	2	+ ·020	−0·015
2·0 to 2·9	3	+ ·040	+ ·005
3·0 to 3·9	10	+ ·036	+ ·001
4·0 to 4·4	12	+ ·029	− ·006
4·5 to 4·9	27	+ ·043	+ ·008
5·0 to 5·4	31	+ ·021	− ·014
5·5 to 5·9	60	+ ·033	− ·002
6·0 to 6·4	33	+ ·043	+ ·008
6·5 to 6·9	19	+ ·036	+ ·001
7·0 to 7·9	19	+ ·034	− ·001
8·0 and over	7	+0·019	− ·016

Whence it would appear that the variation of personality with magnitude as far as magnitude 8·0 is sensibly the same at Cambridge as at Greenwich; that is, bright stars are observed both at Cambridge and Greenwich relatively too early by $0^{s}{\cdot}023$ per magnitude.

9. Now this result is in satisfactory agreement with the results of experiments made with screens at Greenwich about 1891 (see a paper by the Astronomer Royal: *Preliminary Note on change of Personal Equation with Stellar Magnitude in Transits observed with the Transit Circle at the Royal Observatory, Greenwich, Monthly Notices*, vol. li. p. 455). The transit of a star when its apparent magnitude was diminished by covering the object-glass with a screen, was found to be recorded later by the following quantities (p. 458) :—

	s
Screen *a*	+0·031
Screen *b*	+0·026
Screen *c*	+0·035
Mean	+0·031

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But it was found rather difficult to estimate the photometric value of the screen. "We may say . . . . that the screens *a*, *b*, and *c* each diminish the brightness by between 1·5 and 2·0 magnitudes," is the cautious conclusion arrived at (p. 456). Thus the change of personal equation per magnitude lies between $0^{s}\cdot021$ and $0^{s}\cdot016$. The value of this quantity found in the present paper ($0^{s}\cdot023$) slightly exceeds the former limit, but there is no serious discrepancy between the results; and when the difficulties of the "screen" method are considered, we cannot but feel that the results arrived at by its use, both at Berlin and at Greenwich, are surprisingly good.

10. Let us glance for a moment at the sort of systematic error this implies, for instance, in the Greenwich clock-star list. The mean magnitude of the stars in the list is about 3·8; and the mean excess of magnitude for each hour of the twenty-four is shown below, with the corresponding correction to the mean hourly R.A., on the assumption of a systematic error of $0^{s}\cdot023$ per magnitude:—

Hour.	Excess of Mag.	Correction.	Hour.	Excess of Mag.	Correction.	Hour.	Excess of Mag.	Correction.
		s			s			s
0–1	+0·3	−·007	8–9	+0·6	−·014	16–17	−0·7	+·016
1–2	−0·1	+·002	9–10	+0·5	−·012	17–18	−0·3	+·007
2–3	+0·3	−·007	10–11	+0·5	−·012	18–19	−0·3	+·007
3–4	+0·5	−·012	11–12	0·0	·000	19–20	+0·3	−·007
4–5	−0·1	+·002	12–13	+0·3	−·007	20–21	+0·4	−·009
5–6	−1·5	+·030	13–14	0·0	·000	21–22	0·0	·000
6–7	−0·4	+·009	14–15	+0·2	−·005	22–23	−0·2	+·005
7–8	−0·8	+·018	15–16	+0·1	−·002	23–24	+0·6	−·014

Of course, so long as the habits of the observers remain much the same, and the clock-star list is not altered, these systematic errors do not appear. We may replace the bright stars (taking faint stars as standard) by fictitious stars of slightly less R.A., and consider the observations always made on the fictitious stars. How far the habits of observers remain constant is yet to be determined. There is a good deal of straightforward work ready for anyone who will undertake it in the comparison of catalogues according to magnitude. I must confess that the above comparison of Cambridge and Greenwich, and the similarity of habit thereby indicated, came as a surprise to me when I found how large the magnitude equation at Cambridge was. I quite expected a different magnitude equation at Greenwich, if only because the chronographic method is used, instead of the eye-and-ear method as at Cambridge. If this personality turns out to be even approximately the same at a number of observatories it will be something to be thankful for.

As regards the clock-star list, it has been much extended since Pond's days. Are some of the systematic errors of his list,

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hitherto supposed to have arisen from his observing too short groups of clock-stars, really due in part to this magnitude personality? The quantities given in the above table are in some respects not unlike those shown in the Introduction to the Nine-Year Catalogue (1872·0).

11. Returning to the actual Cambridge personality, one important question is—Does it vary with R.A.? The answer seems to be in the affirmative; but the variation is small compared with the whole amount. It is also rather exceptional in character; and as yet I can suggest no physical reason for it. The personality is rather greater for 21^{h}–3^{h} R.A. and for 12^{h}–15^{h} R.A. than for the rest of the circuit, and is greater proportionately, *i.e.* the increase is greatest for bright stars.

12. Before giving the numbers which seem to show this we must remove a source of error. The residuals which are here under discussion represent the differences between individual stars and the mean star on each plate. Now the magnitude of this mean star varies; for the whole Cambridge Catalogue it has the following values:—

Mean Magnitude of Stars in the Cambridge Catalogue.

R.A.							
0^{h}–3^{h}.	3^{h}–6^{h}.	6^{h}–9^{h}.	9^{h}–12^{h}.	12^{h}–15^{h}.	15^{h}–18^{h}.	18^{h}–21^{h}.	21^{h}–0^{h}.
8·84	8·75	8·82	8·79	8·70	8·84	9·00	8·92

and we must accordingly apply to the three-hour groups of individual stars the following corrections:—

$-0^{s}\cdot001$ $+0^{s}\cdot006$ $0^{s}\cdot000$ $+0^{s}\cdot003$ $+0^{s}\cdot010$ $-0^{s}\cdot001$ $+0^{s}\cdot013$ $-0^{s}\cdot007$

which have been applied in what follows.

13. Collecting, then, the residuals into three-hour groups, we get the following differences from the mean for that particular magnitude:—

	R.A.							
Mag.	0^{h}–3^{h}. s	3^{h}–6^{h}. s	6^{h}–9^{h}. s	9^{h}–12^{h}. s	12^{h}–15^{h}. s	15^{h}–18^{h}. s	18^{h}–21^{h}. s	21^{h}–0^{h}. s
5·0 to 6·9	+·030	−·005	−·015	−·013	+·010	−·005	−·024	+·011
7·0 to 7·9	+·021	−·014	+·004	−·006	+·008	−·019	−·006	+·004
8·0 to 8·4	+·010	−·003	·000	+·001	...	...	...	...
8·5 to 8·9	+·006	+·004	·000	−·004	...	...	...	...
9·0 to 9·4	−·010	+·006	−·002	+·007	...	...	...	...

Beyond magnitude 7·9 the analysis has only been carried as far as 12^{h}; but it is clear that the discrepancy is greatest for the brightest stars.

This is perhaps more obvious in the following short table:—

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Mag.	Mean Excess.	
	0^h-3^h over 3^h-12^h.	12^h-15^h and 21^h to 0^h over 15^h-21^h.
	s	s
5·0 to 6·9	+ ·041	+ ·025
7·0 to 7·9	+ ·026	+ ·019
8·0 to 8·4	+ ·011	...
8·5 to 8·9	+ ·006	...
9·0 to 9·4	− ·014	...

The point requires further examination, but it seems well to draw attention to it.

14. When all the Oxford plates are measured groups will be formed for each tenth of a magnitude. This has been done to a considerable extent at present, but nothing important is thereby added to the main results above given. Up to magnitude 8·0 the stars are not numerous enough; the accidental errors merely mask the general form of the curve. Beyond 8·0 the groups for each tenth show rather more clearly the transition from the gradient for bright stars to that for faint stars. As remarked above, only the first 12^h of R.A. have yet been examined. The results are given below for magnitudes 7·0 to 9·5 :—

Mag.	No. Stars.	Personality.	Mag.	No. Stars.	Personality.	Mag.	No. Stars.	Personality.
		s			s			s
7·0	131	+ ·101	7·9	78	+ ·051	8·8	244	+ ·015
7·1	30	+ ·097	8·0	139	+ ·057	8·9	331	+ ·013
7·2	56	+ ·084	8·1	79	+ ·046	9·0	710	− ·005
7·3	55	+ ·066	8·2	109	+ ·059	9·1	388	− ·033
7·4	48	+ ·048	8·3	108	+ ·044	9·2	292	− ·057
7·5	156	+ ·073	8·4	137	+ ·042	9·3	321	− ·062
7·6	64	+ ·095	8·5	250	+ ·040	9·4	290	− ·079
7·7	95	+ ·066	8·6	164	+ ·020	9·5	438	− ·066
7·8	111	+ ·075	8·7	237	+ ·026			

Summary.

1. The Cambridge meridian observations show a distinct systematic "magnitude equation" or personality depending on the brightness of the star.

2. In R.A. this may be described by saying that to magnitude 8·0 or thereabouts bright stars are observed too early, by $0^s·023$ per magnitude. For stars fainter than 8·0 the personality increases rapidly to $0^s·100$ per magnitude.

3. In declination the personality does not appear till about magnitude 7·5; stars fainter than this are observed too far south.

4. Comparison with Berlin observations shows that the magnitude-equation determined by Dr. Becker from observations

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with screens ($0^s\cdot007$ per magnitude) is confirmed, except that this law changes for stars fainter than 8·5, which he perhaps did not sufficiently observe with the screens.

5. Comparison with Greenwich observations shows that the magnitude-equations at Cambridge and Greenwich are sensibly the same to magnitude 8·0—a result in fair accordance with observations made at Greenwich with screens in 1891.

6. The magnitude-equation at Cambridge is sensibly greater for stars of R.A.'s 21^h–3^h and 12^h–15^h than for stars in R.A.'s 3^h–12^h and 15^h–21^h.

Note added 1899 November 10: Since writing the above paper I have this morning seen the Cape Catalogue for 1890, just published, in the introduction to which Dr. Gill compares the catalogue according to stellar magnitude with other catalogues, and concludes that "these facts prove :—

"(1) The reality of the corrections derived by Auwers.

"(2) That the average observer is affected by a fairly constant personal error depending on magnitude.

"(3) The efficiency of the screen method for determining this error."

The first point is amply confirmed by the results of the present paper.

No. (2) confirms and extends the conclusion arrived at in the present paper by a comparison of the Cambridge (1875) and Greenwich (1880) Catalogues : that the magnitude-equation is often (perhaps generally) the same for different observers so far as magnitude 8·0 or thereabouts.

But with regard to (3) I think the present paper shows that :—

(A) Photographic measures give a much better method for determining the magnitude-equation.

(B) The screen method is liable to under-estimate the amount of the equation. Dr. Gill finds it by the screen method to be $0^s\cdot014$ per magnitude. We have found above that it is $0^s\cdot023$ for Greenwich, and so probably for the Cape also. The "screen" method used at Greenwich and Berlin gave similarly a small coefficient. The difficulty is in estimating the photometric value of the screen. The screens used at Greenwich were originally supposed to represent two magnitudes ; and if that were so the resulting effect on observations of R.A. $0^s\cdot031$ represents $0^s\cdot016$ per magnitude, which is sensibly Dr. Gill's result. But during the actual observations at Greenwich reasons were found for suspecting this assumed photometric value ; and this is expressed in the Astronomer Royal's paper by the cautious conclusion above quoted : "We may say that the screens *a*, *b*, and *c* each diminish the brightness by between 1·5 and 2·0 magnitudes." It would seem that even the lower limit is not quite low enough. The result $0^s\cdot031$ corresponds to 1·3 magnitudes as the photometric value of the screen ; and it would seem that the

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observations made rather determine this photometric value than the amount of the magnitude-equation. Dr. Gill has begun measuring star photographs with a very accurate micrometer. I cannot help thinking that if he will take a few plates near the equator, and measure them for comparison with his meridian observations, he will quickly and easily determine the magnitude-equation and find reason to revise his opinion of the value of the "screen" method. In any case the independent check would be very valuable.

On the Distribution of Stars Photographed at the University Observatory, Oxford, for the Astrographic Catalogue for Zones +25° to +29°. By F. A. Bellamy.

In *The Observatory* for 1899 July the Astronomer Royal gave a table of the comparison of the number of stars in the Greenwich Astrographic Catalogue, zones +64° to +70°, with those in B.D. and in the A.G.C. (Helsingfors and Christiania). The total number of stars is given for each hour of R.A. (58,170 stars for the twenty-four hours), the number per square degree—the mean being 70·0—the number in B.D. and in A.G.C., and the ratios to Greenwich measures; these ratios are 5·8 and 11·7 respectively. It should be remarked that all the plates had exposures of 6^{m}, 3^{m}, and 20^{s}; that all the plates not showing—with the shortest exposure—stars of the 9·0 mag. on Argelander's scale were rejected; and, thirdly, by the method adopted for measurement, which avoids overlap of adjacent plates, no stars are counted more than once: that article suggested a similar discussion of the Oxford measures.

Firstly, for reasons which need not be entered into here,* the Oxford plates have not all been taken with the uniform exposures of 6^{m}, 3^{m}, and 20^{s}, but these only will be used in this discussion; secondly, the plates are not so consecutive or contiguous, though as numerous, as those used in forming the table for Greenwich; and, thirdly, the basis for the rejection of a plate was not the visibility of the ninth magnitude stars, but the criterion of having three times the number of stars counted within the same area on Argelander's charts, slight allowance being sometimes made if the plate had already been measured and the total number of stars came near the limit. Table I. shows that an average of 4·0 times B.D. stars has been maintained, and 7·4 times the stars in the Cambridge Catalogue.

In spite of these differences the following table may be regarded as fairly comparable with that for Greenwich: the numbers for Greenwich are not repeated, but may be gathered from the Plate 1 accompanying this table.

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* *23rd Report to the Board of Visitors, University Observatory, Oxford*, p. 7.